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ADAPTIVE FILTERING WITH CORRELATED STATE NOISE

P. ARGENTIERO

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ABSTRACT

An adaptive filter which uses a minimum variance criteria to estimate state noise covariance is developed. It is not necessary to assume white state noise in order to implement the filter. Simulation results are given which demonstrate that the filter tracks a satellite in the presence of modeling errors better than a conventional minimum variance filter with state noise. It is also shown that the propagated covariance matrix of the filter is an accurate indicator of the filter's performance.

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ADAPTIVE FILTERING WITH CORRELATED STATE NOISE

INTRODUCTION

The tendency of recursive minimum variance filters to diverge, that is, to show a secular growth of residuals, has received much attention (1-5). The difficulty appears to be rooted in the assumption that the dynamics governing state evolution are precisely known and are properly modeled in the filter. Under such an assumption the state estimate of the filter must ultimately become effectively independent of incoming data. At that point the filter becomes a propagator of a state by means of a dynamic state equation. This causes no difficulties if the state equation be accurate since presumably this point is reached only after the filter has well identified the state. But if, as is usually the case in applications, the state equation is not entirely accurate the tendency of the filter to become independent of incoming data must lead to a growing difference between the actual state and the estimated state. The divergence between true and estimated state will manifest itself in the before mentioned growth of residuals.

One way of taking into proper account inaccuracies in the state equation is to add a random component to the equation thus making the state equation stochastic in nature. The variance of the random component is a measure of the quality of the dynamic modeling at any given time. With such a change the recursive minimum variance filter never loses its responsiveness to incoming data and hence the tendency to diverge is greatly mitigated or eliminated.

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The random component of the state equation is commonly called "state noise" and choosing a-priori the proper variance for the state noise can be a problem. If the state noise variance is too large the filter will be too responsive to data noise and will not perform as well as it would if state noise were properly chosen. Conversely, if the state noise variance is fixed at too small a value, filter divergence may again occur. It is not always an easy matter to determine a-priori the quality of one's dynamic modeling and of course the modeling quality may vary considerably during the course of the filter's performance.

An interesting approach to this problem introduced by A. Jazwinski (6, 7) is to permit the filter a feedback mechanism by which it can continuously monitor its performance, increasing the variance of state noise when residuals indicate that dynamic modeling is poor and reducing the variance when residuals indicate the converse. This technique is called "adaptive filtering." Jazwinski's approach amounts to choosing the variance of the state noise at any given step according to a maximum likelihood criteria based on the sum of a given number of previous residuals. The technique has been demonstrated to be effective in satellite orbit estimation (8). However, there are some objections which can be raised. First, Jazwinski presents his filter algorithm in a form which assumes that the observations are scalars. While it is true that technically this presents no loss of generality it is not always convenient to present observations to a filter in scalar form. Second, the filter is not completely recursive since residuals from several steps back must be stored in the computer. If

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observations are not scalars this can in some cases cause storage and time problems. This factor is also important with regard to possible applications of adaptive filters to on-board, real time estimation. Third, the variance of the state noise in the maximum likelihood approach is conditioned equally by a certain number of previous residuals and has no direct dependence on residuals obtained previous to a certain time. Intuitively one expects that the estimate of state noise variance would be a gradually attenuating function of previous residuals. Such an attenuation could always be enforced on a more or less ad hoc basis but one would prefer that this attenuation occur as a logical result of one's theoretical model. The fourth objective is more subtle than the previous three. Methodologically it is sound to model errors in the dynamics of one's system as a random component of the state equation. What cannot be modeled deterministically can and should be modeled stochastically. What is questionable, however, is Jazwinski's assumption that the state noise is a white random sequence. If dynamic modeling errors have long periods relative to the data sampling rate their net effect will not manifest itself as a realization of a white random sequence. In fact, in most applications it is much more reasonable to model dynamic system errors as a highly colored random sequence. The assumption of white state noise is not necessary for the implementation of Jazwinski's maximum likelihood approach but unless it is imposed a numerically intractible algorithm results.

In this paper the author has taken Jazwinski's valuable idea of adaptive filtering and has cast it into a different mathematical setting in which state noise variance is estimated according to a minimum variance rather than a maximum likelihood criteria. It will be seen that this new approach adequately meets every one of the objections raised against the maximum likelihood approach.

There is an aspect to adaptive filtering which has not received attention. A common criticism against minimum variance filters either in the recursive or in the batch mode is that the related covariance matrix which the filter generates tends to give an optimistic assessment of the filter's performance as measured by residuals. An adaptive filter either from a minimum variance or a maximum likelihood criteria forces a statistical compatibility between residuals and state noise variance and thus indirectly a compatibility is forced between the actual filter errors and the propagated covariance matrix of the filter. A general proof of this claim is a difficult matter since adaptive filters are non linear. Nevertheless, in every one of the numerous simulations the author has performed with his version of the adaptive filter, the actual errors of the filter were statistically compatible with the R.M.S. errors propagated by the filter.

As a final comment before the mathematics of the minimum variance adaptive filter is given it should be mentioned that under no circumstances should adaptive filtering be thought of as an adequate substitute for good modeling. Of course, the best of all possible solutions to the problem of filter divergence is

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improved modeling. This improvement can take the form of a more astute representation of the forces acting on the system or the improvement can be obtained by increasing the size of the estimated state to include a representation of unmodeled forces as Jazwinski has done recently (9). There is a limit, however, to how well one can deterministically model the dynamics of a system. What must be left out can only be accounted for in a stochastic fashion. In what follows it should be assumed that the analysis begins only when all efforts to improve modeling have ended.

A MINIMUM VARIANCE FILTER WITH COLORED STATE NOISE

Assume that the dynamic equation for a time evolving state is

$$\widetilde{X}_n = A_n \widetilde{X}_{n-1} + R_n \tau_n, n = 0, 1, \ldots \tag{1}$$

where $\widetilde{X}_n$ is a K dimensional state, A_n is a K by K state transition matrix, τ_n is an m dimensional random variable and, of course, R_n is of dimension K by m. The statistical properties of τ_n are

$$E(\tau_n) = 0, E(\tau_n \tau_n^T) = \rho_n \tag{2}$$

and

$$\tau_n = \gamma_n \tau_{n-1} + \psi_n \tag{3}$$

where

$$E(\psi_n) = 0, E(\psi_n \psi_n^T) = \phi_n$$

Assume also that there exists an L dimensional observation state $\widetilde{Y}_n$ related to $\widetilde{X}_n$ by

$$\widetilde{Y}_n = B_n \widetilde{X}_n \tag{4}$$

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Let Y_n be a direct measurement of $\tilde{Y}_n$. Thus

$$Y_n = \tilde{Y}_n + \nu_n, E(\nu_n) = 0, E(\nu_n \nu_n^T) = Q_n, E(\nu_i \nu_j^T) = 0, i \neq j$$

$\{\nu_n\}$ and $\{\tau_n\}$ are uncorrelated and all random variables are assumed normally distributed.

Finally one assumes the existence of an unbiased estimator $\hat{X}_{n-1}$ of $\tilde{X}_{n-1}$ with properties

$$\hat{X}_{n-1} = \tilde{X}_{n-1} + a_{n-1}, E(a_{n-1}) = 0, E(a_{n-1} a_{n-1}^T) = P_{n-1} \tag{5}$$

And since presumably a_{n-1} is conditioned by τ_{n-1} a correlation between τ_n and a_{n-1} must be expected. Thus

$$\chi_n = E[\tau_n a_{n-1}^T] \tag{6}$$

A recursion relation for χ_n will be obtained later.

Under the assumption that the covariance matrix of τ_n is known the linear, unbiased, minimum variance filter is known to be

$$\hat{X}_n = A_n \hat{X}_{n-1} + H_n (Y_n - B_n A_n \hat{X}_{n-1}) \tag{7}$$

(where in equation 7 and hereafter we drop the logically correct but inconvenient distinction between random variables and their realizations) with

$$H_n = \delta_1 B_n^T \delta_2^{-1}$$
$$P_n = \delta_1 [I - B_n^T \delta_2^{-1} B_n \delta_1] \tag{8}$$

where

$$\delta_1 = A_n P_{n-1} A_n^T + R_n \rho_n R_n^T - R_n \chi_n A_n^T - A_n \chi_n^T R_n^T \tag{9}$$

$$\delta_2 = Q_n + B_n A_n P_{n-1} A_n^T B_n^T + B_n R_n \rho_n R_n^T B_n^T - B_n R_n \chi_n A_n^T B_n^T - B_n A_n \chi_n^T R_n^T B_n^T$$

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To complete the representation of the minimum variance filter it is necessary to define recursively χ_n as a function of χ_{n-1}. Assuming that $\hat{X}_{n-1}$ can be written as

$$\hat{X}_{n-1} = A_{n-1} \hat{X}_{n-2} + H_{n-1} (Y_{n-1} - B_{n-1} A_{n-1} \hat{X}_{n-2}) \tag{10}$$

a_{n-1} can be represented as

$$a_{n-1} = (A_{n-1} - H_{n-1} B_{n-1} A_{n-1}) a_{n-2} + (H_{n-1} B_{n-1} R_{n-1} - R_{n-1}) \tau_{n-1} + H_{n-1} \nu_{n-1} \tag{11}$$

Equation 11 in conjunction with equation 3 permits us to write

$$\chi_n = \gamma_n \chi_{n-1} (A_{n-1} - H_{n-1} B_{n-1} A_{n-1})^T + \gamma_n \rho_{n-1} (H_{n-1} B_{n-1} A_{n-1} - R_{n-1})^T \tag{12}$$

Equations 7, 8, 9 and 12 define a linear, unbiased, minimum variance filter under the assumption that the covariance matrix of the state noise is known.

ADAPTIVE ESTIMATION OF STATE NOISE COVARIANCE

The state noise covariance is in a sense a measure of dynamic modeling accuracy, and to assume that this matrix is known at every step of the filter's operation is to assume that one knows the quality of his modeling at all times. Often this is not a valid assumption and it is suggested that a feedback mechanism be provided to choose a state noise covariance based on the filter's past performance as measured by residuals. It is the function of this section to establish this mechanism.

First it is convenient to introduce some non-standard matrix operations. Let A be any square matrix of dimension N by N. By the symbol D (A) we shall refer to the N by 1 vector consisting of the diagonal elements of A. Conversely,

let C be an N by 1 vector. Then $D^{-1}(C)$ will represent the N by N matrix whose diagonal elements are the elements of C and whose off diagonal elements are zero. Finally, let A be any N by M matrix. The symbol A^2 will represent an N by M matrix whose elements are the squares of corresponding elements of A.

Attention is focused on equation 3. If each element of τ_n can be considered as an independent unmodeled force it is reasonable to expect that ρ_n be a diagonal matrix. This suggests that we assume γ_n and ϕ_n to be diagonal matrices. Letting

$$\tilde{Z}_n = D(\rho_n)$$

$$\tilde{Z}_{n-1} = D(\rho_{n-1})$$

$$\delta_n = D(\phi_n)$$

One can immediately derive the result

$$\tilde{Z}_n = \gamma \gamma^T \tilde{Z}_{n-1} + \delta_n \tag{13}$$

Equation 13 resembles a state transition equation with $\tilde{Z}_n$ the time evolving state. The resemblance to equation 1 can be strengthened if one recognizes that in applications, equation 13 is only an approximate representation of the time evolution of the state noise covariance and that a random component should be added. We will use as a state equation for $\tilde{Z}_n$

$$\tilde{Z}_n = \gamma \gamma^T \tilde{Z}_{n-1} + \delta_n + \pi_n, \; E(\pi_n) = 0, \; E(\pi_n \pi_n^T) = \theta_n \tag{14}$$

If an observation equation could be obtained to accompany equation 14, it would be possible to estimate $\tilde{Z}_n$ in a fashion analogous to how the state $\tilde{X}_n$ is estimated.

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The observation state should be related to the residuals of the filter. The residual at the nth step is

$$Y_n^R = Y_n - B_n A_n \hat{X}_{n-1} = \nu_n - B_n A_n a_{n-1} + B_n R_n \tau_n \tag{15}$$

and

$$E\left[Y_N^R \left(Y_N^R\right)^T\right] = Q_n + B_n A_n P_{n-1} A_n^T B_n^T$$
$$+ B_n R_n \rho_n R_n^T B_n^T - B_n \chi_n A_n^T B_n^T - B_n A_n \chi_n^T B_n^T \tag{16}$$

Let

$$\tilde{Y}_n = D\left[E\left[Y_N^R \left(Y_N^R\right)^T\right]\right]$$

and

$$\omega_n = D\left[Q_n + B_n A_n P_{n-1} A_n^T B_n^T - B_n \chi_n A_n^T B_n^T - B_n A_n \chi_n^T B_n^T\right] \tag{17}$$

Then one can write

$$\tilde{Y}_n = [B_n R_n]^2 \tilde{Z}_n + \omega_n \tag{18}$$

Equation 18 has the form of a linear observation equation. All that is needed in order to estimate $\tilde{Z}_n$ in a minimum variance fashion is to obtain an unbiased estimator of $\tilde{Y}_n$ whose covariance matrix is available. For $I \leqslant K$, Y_N^R (I) is a scalar sample from a zero mean normal distribution whose variance is $\tilde{Y}_n$ (I). Consequently $(Y_N^R (I))^2 / \tilde{Y}_n$ (I) is the square of a scalar sample from a normal distribution of mean 0 and variance 1. As such it is a sample from a chi-square distribution with degree of freedom 1. The chi-square distribution with 1 degree of freedom has mean 1 and variance 2. From these facts it follows that

$$E\left[\left(Y_N^R\right)^2\right] = \tilde{Y}_n \tag{19}$$

and

$$D\left[E\left[\left(Y_N^R\right)^2 \left(Y_N^R\right)^{T2}\right]\right] = 2\tilde{Y}_n^2 \tag{20}$$

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Equations 19 and 20 assert that $(Y_N^R)^2$ is an unbiased estimator of $\tilde{Y}_n$ the

diagonal elements of whose covariance matrix are given by $2\,\tilde{Y}_n^2$. The off diag-

onal elements are not readily obtainable and must be assumed zero. Another

difficulty is that $\tilde{Y}_n$ is dependent on $\tilde{Z}_n$ which is what we are attempting to esti-

mate. The covariance matrix of $(Y_N^R)^2$ must be estimated by using the best esti-

mate of $\tilde{Z}_n$ as obtained by equation 13. Given that an estimate $\hat{Z}_{n-1}$ of $\tilde{Z}_{n-1}$ is

available our estimate of the covariance matrix of $(Y_N^R)^2$ must be

$$E\left[(Y_N^R)^2\,(Y_N^R)^{T\,2}\right] \approx q_n = 2\,D^{-1}\left[(B_n\,R_n)^2\,[\gamma_n\,\gamma_n^T\,\hat{Z}_{n-1}+\delta_n] + \omega_n\right] \quad (21)$$

The vector $(Y_N^R)^2$ can now be treated as an observation and processed in a mini-

mum variance fashion to yield an estimate of $\tilde{Z}_n$.

Given an unbiased estimator $\hat{Z}_{n-1}$ of $\tilde{Z}_{n-1}$ and its covariance matrix P_{n-1},

and Y_n as defined by equation 15, the following recursion relations define the

minimum variance estimate of $\tilde{Z}_n$.

$$\hat{Z}_n^- = \gamma\,\gamma^T\,\hat{Z}_{n-1} + \delta_n$$

$$Y_n = (Y_n^R)^2$$

$$Y_n^c = [B_n\,R_n]^2\,\hat{Z}_n^- + \omega_n$$

$$\kappa_1 = \gamma\,\gamma^T\,P_{n-1}\,\gamma\,\gamma^T + \theta_n \qquad\qquad (22)$$

$$\kappa_2 = q_n + [B_n\,R_n]^2\,\gamma\,\gamma^T\,P_{n-1}\,\gamma\,\gamma^T\,[R_n^T\,B_n^T]^2 + [B_n\,R_n]^2\,\theta_n\,[R_n^T\,B_n^T]^2$$

$$H_n = \kappa_1\,[R_n^T\,B_n^T]^2\,\kappa_2^{-1}$$

$$\hat{Z}_n = \hat{Z}_n^- + H_n\,[Y_n - Y_n^c]$$

$$P_n = \kappa_1\left[I - [R_n^T\,B_n^T]^2\,\kappa_2^{-1}\,[B_n\,R_n]^2\,\kappa_1^T\right]$$

The filter is recursive in the sense that only the present residual must be available, the residual is not assumed to be scalar, $\hat{Z}_n$ is an automatically attenuating function of previous residuals and it is not necessary to assume the state noise to be white. As promised, all objections to maximum likelihood adaptive filtering are adequately met by the minimum variance adaptive filter.

It is instructive to see explicitly how the minimum variance adaptive feature interfaces with the minimum variance filter presented by equations 7 through 12. Given the mathematical model of equations 1 through 5 and given

$$\hat{X}_{n-1}, P_{n-1}, Y_n, \chi_{n-1}, P_{n-1}, \rho_{n-1}, H_{n-1}, A_{n-1}, B_{n-1},$$

the full set of recursion relations for the minimum variance adaptive filter is

$$\hat{X}_n^- = A_n \hat{X}_{n-1} \tag{23a}$$

$$Y_n^R = Y_n - B_n \hat{X}_n^- \tag{23b}$$

$$\chi_n = \gamma_n \chi_{n-1} (A_{n-1} - H_{n-1} B_{n-1} A_{n-1})^T + \gamma_n \rho_{n-1} (H_{n-1} B_{n-1} A_{n-1} - R_{n-1})^T \tag{23c}$$

$$\hat{Z}_{n-1} = D(\rho_{n-1}) \tag{23d}$$

$$\hat{Z}_n^- = \gamma \gamma^T \hat{Z}_{n-1} + \delta_n \tag{23e}$$

$$Y_n = (Y_N^R)^2 \tag{23f}$$

$$\omega_n = D[Q_n + B_n A_n P_{n-1} A_n^T B_n^T - B_n \chi_n A_n^T B_n^T - B_n A_n \chi_n^T B_n^T] \tag{23g}$$

$$Y_n^c = [B_n R_n]^2 \hat{Z}_n^- + \omega_n \tag{23h}$$

$$\kappa_1 = \gamma_n \gamma_n^T P_{n-1} \gamma_n \gamma_n^T + \theta_n \tag{23i}$$

$$q_n = 2 D^{-1} \left[[B_n R_n]^2 [\gamma_n \gamma_n^T \hat{Z}_{n-1} + \delta_n] + \omega_n \right] \tag{23j}$$

$$\kappa_2 = q_n + [B_n R_n]^2 \gamma_n \gamma_n^T P_{n-1} \gamma_n \gamma_n^T [R_n^T B_n^T]^2 + [B_n R_n]^2 \theta_n [R_n^T B_n^T]^2 \tag{23k}$$

$$H_n = \kappa_1 [R_n^T B_n^T]^2 \kappa_2^{-1} \tag{23l}$$

$$\hat{Z}_n = \hat{Z}_n^- + H_n \, [Y_n - Y_n^c] \tag{23m}$$

$$P_n = \kappa_1 \left[I - (R_n^T \, B_n^T)^2 \, \kappa_2^{-1} \, [B_n \, R_n]^2 \, \kappa_1^T \right] \tag{23n}$$

$$\rho_n = D^{-1} (\hat{Z}_n) \tag{23o}$$

$$\delta_1 = A_n \, P_{n-1} \, A_n^T + R_n \, \rho_n \, R_n^T - R_n \, \chi_n \, A_n^T - A_n \, \chi_n^T \, R_n^T \tag{23p}$$

$$\delta_2 = Q_n + B_n \, A_n \, P_{n-1} \, A_n^T \, B_n^T + B_n \, R_n \, \rho_n \, R_n^T \, B_n^T - B_n \, R_n \, \chi_n \, A_n^T \, B_n^T - B_n \, A_n \, \chi_n^T \, R_n^T \, B_n^T \tag{23q}$$

$$H_n = \delta_1 \, B_n^T \, \delta_2^{-1} \tag{23r}$$

$$\hat{X}_n = \hat{X}_n^- + H_n \, Y_n^R \tag{23s}$$

$$P_n = \delta_1 \, [I - B_n^T \, \delta_2^{-1} \, B_n \, \delta_1] \tag{23t}$$

The structure of this elaborate set of recursion relations may become clearer if one views equation 23 as a pair of interlocking minimum variance filters. The inner filter defined by equations d through o estimates state noise covariance and the outer filter defined by equations a through c and p through t estimates the state $\tilde{X}_n$. The outer filter passes the residual Y_N^R and χ_n to the inner filter which processes $(Y_N^R)^2$ as an observation and estimates ρ_n. The matrix ρ_n is treated by the outer filter as the state noise covariance and an estimate $\hat{X}_n$ of $\tilde{X}_n$ is obtained.

One has several controls on the behavior of the inner filter. The diagonal matrix γ_n is chosen on the basis of the expected relationship between sampling rate and periodicities in unmodeled forces. The vector δ_n is then chosen by specifying the expected average level of state noise for each unmodeled force. If the average level of state noise variance for the ith unmodeled force is χ_i,

then in order for the asymptotic value of the ith component of $\widetilde{Z}_n$ as generated by equation 13 to equal X_i, one must have

$$X_i = \delta_n(i)/1 - \delta^2(i, i) \qquad (24)$$

Equation 24 explicitly specifies the elements of δ_n. The matrix θ_n determines the responsiveness of the inner filter's estimate of state noise covariance to the present residual. If the eigenvalues of θ_n are large, two or three large residuals in succession will drastically change the inner filter's estimate of state noise covariance. This will cause the outer filter to be guided less by modeling and more by incoming data. Consequently if modeling errors are expected to have a sudden onset a matrix θ_n with large eigenvalues is desirable. Otherwise such a matrix is undesirable since it forces the filter to interpret a small number of residuals as indicating the presence of a modeling error which may not exist.

SIMULATIONS

The non-linear nature of adaptive filters make performance evaluation by means of ensemble studies impossible. If one is to know the capabilities of an adaptive filter in a given situation, simulations must be performed. This simulation will compare the performance of the minimum variance adaptive filter as presented in previous sections with that of a standard minimum variance filter with state noise covariance chosen a-priori. Both filters will employ simple two-body theory on an earth orbit which initially is approximately circular with

a 8,000 km radius. In both filters, the R_n matrix of equation 1 was chosen under the premise that any unmodeled forces are pointing toward the earth's center. This permits the state noise to be modeled as a scalar random variable, and simplifies somewhat the recursion relations of equation 24. The observations were generated under the assumption that in addition to a point mass earth another point mass, a "mass concentration" of considerable size, was situated just under the earth surface in such a way that its impact on satellite dynamics was considerable. A random number generator was used to add the appropriate random component to the true observations. The added point mass was made very large so that substantial efforts could be seen in a short period of time. Each filter's performance at any given time was measured by the distance between the actual position of satellite and the estimated position. Both filters assume initially that the standard deviation of the acceleration due to the unmodeled force is $0.1 \, m/sec^2$. Table 1 provides complete orbital and tracking information for the simulation. Since modeling errors were expected to be long relative to the data sampling rate, γ_n which in this case is a scalar was chosen to be 0.9. The variance θ_n of the state noise of the inner filter state equation was set at $10^{-3} \, m^2/sec^4$. No systematic effort was made to choose these parameters so as to optimize the adaptive filter's performance. It is likely that a more judicious choice of γ_n and θ_n is possible. The results of the comparison are shown in Figure 1. The initial transient errors of both filters are off scale and are not shown. Figure 1 demonstrates that the standard minimum variance filter does

Table 1

Orbital and Tracking Information for Simulation of Figure 1

ORBITAL INFORMATION	
State of Satellite at T_O	in 8,000 km circular, equatorial, orbit, longitude 45°
Size of Mass Concentration	0.5% of earth mass
Location of Mass Concentration	100 km beneath earth surface, longitude 60°, latitude 5°
A-priori Estimate of State Noise Standard Deviation	0.1 m/sec^2

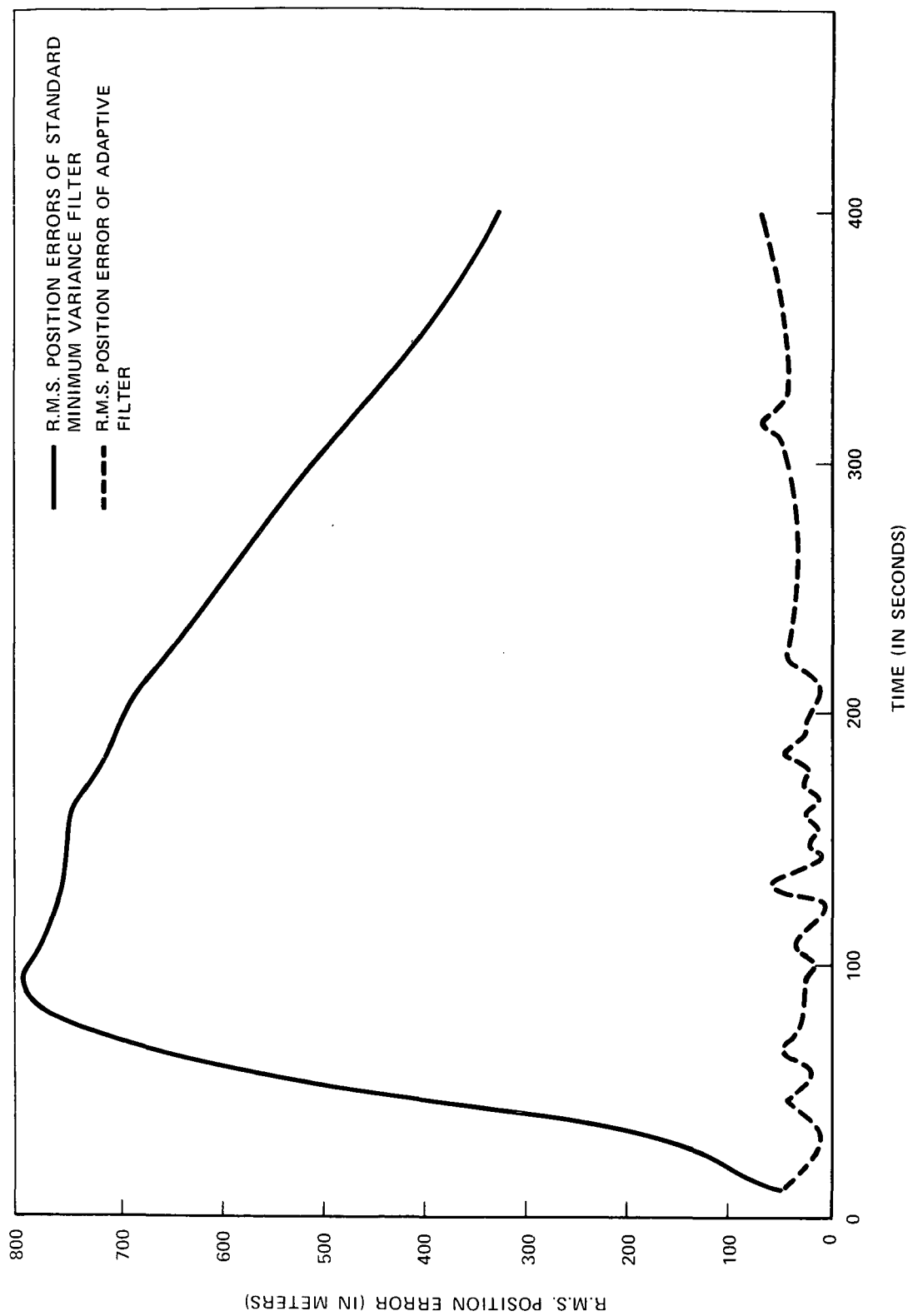| TRACKING INFORMATION						
	Longitude	Latitude	Data Type	Acquisition Rate	Standard Deviation	Observation Period
Tracking Station 1	30°	15°	Ranging	10/min	10 m	T_0 to T_0 + 400 sec
Tracking Station 2	60°	30°	Ranging	10/min	10 m	T_0 to T_0 + 400 sec
Tracking Station 3	65°	-25°	Ranging	10/min	10 m	T_0 to T_0 + 400 sec

Figure 1. Comparison of R.M.S. Position Errors of Adaptive Filter and Standard Minimum Variance Filter

not track well in the presence of an unmodeled mass concentration. As the satellite recedes from the mass concentration its performance gradually improves. The satellite is also receding from the tracking stations with a resultant loss of observability. This accounts for the slow decay of the adaptive filter's performance after about 300 seconds of tracking time. After 400 seconds of tracking time the adaptive filter's estimate of the state is substantially better than that of the standard minimum variance filter. In the region where the unmodeled force had its greatest effect, the inner filter of the adaptive filter was returning estimates of the standard deviation of the state noise between $1\,\text{m/sec}^2$ and $4\,\text{m/sec}^2$. As the satellite receded from the source of the unmodeled force the estimate of the standard deviation of the state noise stabilized at between $0.1\,\text{m/sec}^2$ and $0.5\,\text{m/sec}^2$. The state noise variance used by the standard minimum variance filter was of course fixed at $0.1\,\text{m/sec}^2$. This was not adequate to permit the filter to respond to large residuals caused by the unmodeled force. Had a much larger value been chosen for the standard deviation of state noise, the standard minimum variance filter would have tracked well in the presence of the unmodeled force but it would have behaved erratically when no unmodeled forces were present.

It was mentioned in the introduction that in all the simulations the Author has performed the nominal R.M.S. position errors as obtained from the propagated covariance matrix of the adaptive filter have been statistically compatible with the actual filter errors. Another simulation the details of which are

Table 2

Orbital and Tracking Information for Simulation of Figure 2

ORBITAL INFORMATION	
State of Satellite at T_0	in 10,000 km circular, equatorial, orbit, longitude 45°
Size of Mass Concentration	0.5% of earth mass
Location of Mass Concentration	100 km beneath earth surface, longitude 60°, latitude 5°
A-priori Estimate of State Noise Standard Deviation	0.1 m/sec²

TRACKING INFORMATION						
	Longitude	Latitude	Data Type	Acquisition Rate	Standard Deviation	Observation Period
Tracking Station 1	30°	15°	Ranging	7/min	10 m	T_0 to T_0 + 600 sec
Tracking Station 2	60°	30°	Ranging	7/min	10 m	T_0 to T_0 + 600 sec
Tracking Station 3	65°	5°	Ranging	7/min	10 m	T_0 to T_0 + 600 sec

displayed in Table 2 was performed in order to give a demonstration of this fact. Figure 2 compares the actual R.M.S. position errors of the adaptive filter with its nominal R.M.S. position errors as obtained from its propagated covariance matrix. Again initial values are off scale and are not shown. The two curves cohere until poor observability causes difficulties after about 500 seconds of tracking time. Assuming filter errors are normally distributed they should exceed their R.M.S. values in approximately one-third of the cases. By this criteria the propagated covariance matrix of the adaptive filter at least in this simulation appears to be a slightly conservative estimate of the filter's actual errors.

CONCLUSIONS

This paper has presented a new algorithm for adaptively estimating state noise covariance within the context of a recursive minimum variance filter. The procedure utilizes a minimum variance rather than a maximum likelihood criteria. The state noise is modeled as the resultant of a stochastic linear difference equation, an assumption considerably more general than the usual modeling of state noise as a white random sequence. The algorithm is completely recursive and the state noise covariance estimate is an attenuating function of previous residuals.

The minimum variance adaptive filter has been shown in simulations to be more effective in tracking a satellite in the presence of modeling errors than a
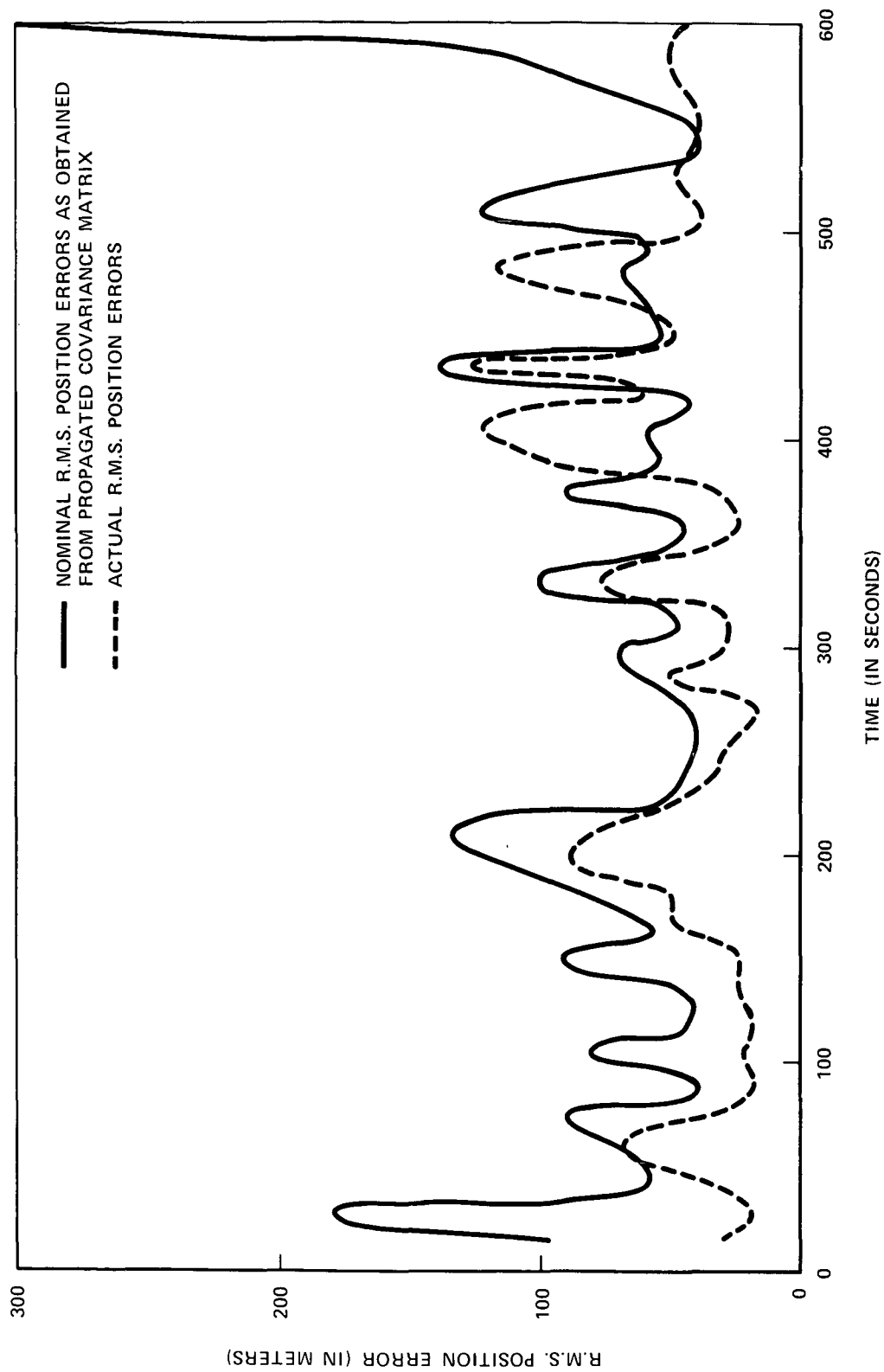
19

Figure 2. Comparison of Actual and Nominal R.M.S. Position Errors of Adaptive Filter

conventional minimum variance filter with state noise fixed a-priori. It was also shown that in contrast to the conventional minimum variance filter the propagated covariance matrix of the minimum variance adaptive filter is an accurate indicator of the actual performance of the filter.

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